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GAME PROBLEM OF THE DESIGN OF A MULTI-IMPULSE MOTION CORRECTION*

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A dynamic system acted on by magnitude-bounded noise is examined. The deviation caused by the noise is corrected by a finite number of impulses whose total resource is bounded. The problem of designing an optimal correction whose elements are certain (signal) surfaces is investigated. The application of the dynamic programming method leads to a boundary-value problem with an unknown boundary. Necessary conditions (relations (2.5)) are obtained for the optimality of the desired boundary (the signal surface). These conditions, together with the well-known dynamic programming relations, form a complete system of necessary conditions permitting the construction of the problem's solution. The approach presented is applied to a two-parameter system admitting of self-similar solutions. In theme the paper border on the investigations in /1—4/; a programmed approach to impulse correction problems was used in /3—5/; /6—9/ contain investigations on the design of optimal correction in stochastic systems.

1. Statement of the problem. Let the motion of a dynamic system on a fixed interval $[t_0, T]$ be described by the differential equation and initial condition

$$x^{\cdot} = f(x, v, t) + B(t)u, \quad t_0 \leqslant t \leqslant T, \quad x(t_0 - 0) = x^0 \tag{1.1}$$

Here $x \in R^n$ is the system's phase vector, $B(t)$ is an $n \times n$-matrix continuous in t, $u \in R^n$ is the control, $v \in R^m$ is the noise vector whose value at instant t has the constraint

$$v(t) \in V_t \subset R^m, \quad m \leqslant n \tag{1.2}$$

where V_t is a convex compact set continuously dependent on time. As admissible noise we consider summable time functions $v(t)$, $t_0 \leqslant t \leqslant T$, satisfying constraint (1.2). We assume that the time realization of control u is a sum of a finite number of delta functions

$$u(t) = \sum_{i=1}^{N} u_i \delta(t - t_i), \; t_0 \leqslant t_1 < \ldots < t_N \leqslant T, \; \sum_{i=1}^{N} R(u_i) \leqslant Q > 0 \tag{1.3}$$

The total number N of impulses and their resource Q are specified. The intensity of an individual impulse is estimated by a continuous scalar function $R(u)$, $R(u) > 0$, $u \in R^n$, $u \neq 0$, $R(0) = 0$.

The control is called on to neutralize the action of the noises in the sense of minimizing the functional

$$J = F(x(T)) \tag{1.4}$$

where $F(x)$ is a prescribed continuous function bounded from below. It is assumed that the function $f(x, v, t)$ in (1.1) is continuous in all the arguments x, v, t and satisfies a Lipschitz condition in x, so that for any admissible noise $v(t)$ and vector $x' \in R^n$ a unique solution of the Cauchy problem /10/

$$x^{\cdot} = f(x, v, t), \quad t' \leqslant t \leqslant T, \quad x(t') = x' \tag{1.5}$$

exists on the interval $[t', T]$, $t_0 \leqslant t' < T$. A motion (solution) of system (1.5) is called an uncontrollable motion of system (1.1). Later on we examine a positional method for forming control u. An arbitrary current state of system (1.1) is completely described by specifying a position (x, q, t, k), $x \in R^n$, $q \in [0, Q]$, $t \in [t_0, T]$, $k = 0, 1, \ldots, N$, where the quantity q has the sense of total intensity on the k impulses allowed. The vector x is, for definiteness, treated as the system's phase state up to a possible impulse at instant t, i.e., $x = x(t - 0)$.

We say that a positional control (a design) has been specified if in the space (x, q, t) signal hypersurfaces Γ_k have been prescribed, which separate the region being considered from this space into two sets G_k and D_k for each $k = 1, \ldots, N$ (Fig.1), and if the functions $u_k = u_k(x, q, t)$ for $(x, q, t) \in D_k$ for each $k = 1, \ldots, N$. Surface Γ_k belongs to the boundary of the closed set D_k. If $(x, q, t) \in D_k$ at the initial instant (point 1 in Fig.1), then the first of the k admissible impulses of intensity $u_k(x, q, t)$ is fed in right away. At the points of set G_k we assign $u_k(x, q, t) = 0$; the impulse is fed in only until the representative point (x, q, t) reaches the signal surface Γ_k (point 2). By definition we set $\Gamma_0 = D_0 = \{(x, q, t): t = T\}$.

The following constraints on the design follow from 1.3:

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$$R(u_k(x, q, t) \leqslant q, \quad k = 1, \ldots, N, \quad (x^+, q^+, t) \in G_{k-1} \cup M_0, \quad M_0 = \{(x, q, t): q = 0\}, \quad k = 2, \ldots, N \qquad (1.6)$$

$$x^+ = x + B(t)u_k, \quad q^+ = q - R(u_k)$$

Here M_0 is a set in space (x, q, t), corresponding to a zero resource. The inclusion in (1.6) ensures that two successive nonzero impulses are spread out in time, i.e., ensures strict inequalities in (1.3). The collection (u_k, Γ_k), $k = 1, \ldots, N$, satisfying constraints (1.6), is called an admissible design and is denoted by u for brevity. We note that the surfaces Γ_k may not be specially delineated, but they can be treated through the agency of functions $u_k(x, q, t)$ as the boundaries of sets on which $u_k = 0$. However, the necessary optimality conditions presented later on in the paper are written in terms of Γ_k.

From the specified admissible design u, noise v and position (x, q, t, k) a unique phase trajectory $x(\tau)$, $\tau \in [t, T]$ of system (1.1) is defined, having no more than $k+1$ intervals of absolute continuity. Let us describe an algorithm for constructing trajectories. If $q = 0$ or $k = 0$, the motion of system (1.1) is uncontrollable and the assertion made is valid. Let $q, k > 0$. We can examine an uncontrollable motion $x(\tau)$, $\tau \in [t, T]$, $x(t) = x$, and find the earliest instant $\tau = t_1 \in [t, T]$ for which the inclusion $x(\tau) \in D_k$ is fulfilled. If $x(\tau) \notin D_k$ obtains for all $\tau \in [t, T]$, the trajectory construction is ended. Otherwise, from the representative point up to the jump, (x_1, q, t_1), $x_1 = x(t_1)$, we construct the representative point after the jump, (x_1^+, q^+, t_1), in accordance with equalities (1.6). If $t_1 = T$, the construction ends and the trajectory is completed by a jump at instant T. In the case $t \leqslant t_1 < T$ we arrive at the original situation but from the position $(x_1^+, q^+, t_1, k-1)$. The procedure is repeated no more than k times and the process ends either with the construction of the last uncontrollable part of the trajectory or with a jump at instant T. In the latter case we should use the vector $x(T+0)$ to compute functional (1.4). Thus, to the specified u and $v(x, q, t, k)$ corresponds a unique trajectory of system (1.1) and, correspondingly, the value $J = J(u, v, x, q, t, k) = F(x(T))$ of functional (1.4).

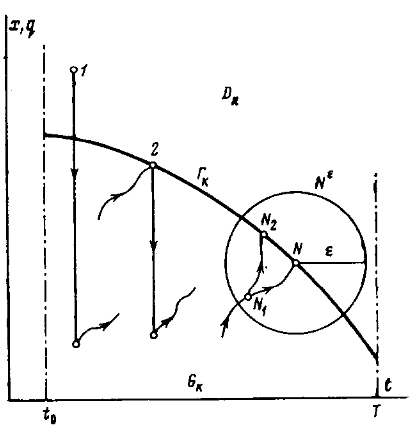

Fig.1

We define the Bellman function $S_k(x, q, t)$, $k = 0, 1, \ldots, N$, $x \in R^n$, $q \in [0, Q]$, $t \in [t_0, T]$ by the equality

$$S_k(x, q, t) = \min_u \sup_v J \qquad (1.7)$$

under the assumption that the minimum exists. The extrema in (1.7) are computed with respect to those parts of the admissible designs and noise that determine the motion from position (x, q, t, k). We continue the functions $S_k(x, q, t)$, $k = 1, \ldots, N$, onto the boundary $t = T$ by continuity and we define there the function S_0 by the equality

$$S_0(x, q, T) = F(x) \qquad (1.8)$$

It can be shown that then the function $S_k(x, q, t)$, $k = 0, 1, \ldots, N$, is continuous for $x \in R^n$, $q \in [0, Q]$, $t \in [t_0, T]$. The magnitude of $S_k(x, q, t)$ is the minimum guaranteed value of functional (1.4) if the motion of system (1.1) starts at instant t from point x and a resource q is prescribed on k correction impulses. From the sense of the problems and from equality (2.1) it follows that the functions $S_k(x, q, t)$ decreases, in general, as q or k grows.

We note that the question on sufficient conditions for the existence of an optimal design for system (1.1)—(1.3) is the subject of a special investigation and is not examined in the present paper.

2. Necessary conditions. We introduce into consideration the functions $S_k^+(x, q, t)$, $x \in R^n$, $q \in [0, Q]$, $t \in [t_0, T]$, setting, by definition,

$$S_k^+(x, q, t) = \min_u S_{k-1}(x + B(t)u, \; q - R(u), \; t), \quad R(u) \leqslant q, \quad k = 1, \ldots, N \qquad (2.1)$$

From the continuity of functions S_k and R follows the continuity of S_k^+ in its domain. Further, let $S(x, t)$ be a scalar function differentiable in x and t. We adopt the notation

$$P(S) = \frac{\partial S}{\partial t} + \max_v \left(\frac{\partial S}{\partial x}, f(x, v, t)\right), \quad v \in V_t \qquad (2.2)$$

in which the scalar product of the gradient vector and the vector of right-hand sides in (1.5) has been indicated by parentheses. We assume that the optimal design u exists, i.e., optimal u_k and Γ_k exist, while in the domains being considered the functions S_k and S_k^+ have nonempty sets of points of continuous differentiability. Then the following system of necessary optimality conditions holds:

$$P(S_k) = 0, \quad (x, q, t) \in G_k, \quad S_k|_{\Gamma_k} = S_k^+, \quad S_0^+|_{\Gamma_0} = F(x), \quad k = 0, 1, \ldots, N \qquad (2.3)$$

$$S_k(x, q, t) = S_k^+(x, q, t), \quad (x, q, t) \in D_k, \quad k = 1, \ldots, N \tag{2.4}$$

$$P(S_k^+) \geqslant 0, \quad (x, q, t) \in D_k, \quad P(S_k^+) \leqslant 0, \quad (x, q, t) \in G_k \cap N^\varepsilon, \quad k = 1, \ldots, N \tag{2.5}$$

The relations containing operator P hold at the points of continuous differentiability of functions S_k and S_k^+. The second inequality in (2.5) is examined in the ε-neighborhood of only those points $N = (x', q', t')$ of boundary Γ_k, for which a nonzero interval $0 < \delta < \delta_0(N)$ of values of δ exists satisfying the condition

$$(x, q, t - \delta) \in G_k, \quad (x, q, t + \delta) \in D_k, \quad (x, q, t) \in \Gamma_k \cap N^\varepsilon \tag{2.6}$$

for some $\varepsilon = \varepsilon(N) > 0$. Condition (2.6) signifies that a straight line parallel to the t-axis in space (x, q, t) strictly intersects surface Γ_k at point N.

Let us turn to the justification of the necessity of conditions (2.3)–(2.5). Equality (2.4) follows from the definitions of functions S_k and S_k^+ and of the domains D_k and expresses the relation of discrete dynamic programming /11/. Conditions (2.3) are dynamic programming relations for a continuous system; they can be obtained by considering in domains G_k the auxiliary controlled process (1.5) with parameter q, termination condition $(x, q, t) \in \Gamma_k$ and functional to be maximized, equal to the magnitude of $S_k^+(x, q, t)$ at the process termination instant. To justify the first inequality in (2.5) we consider an abitrary interior point (x, q, t) of set D_k; here q plays the role of a parameter and, therefore, the limit values 0 and Q can be allowed for it. From the optimality of the design being examined it follows that for any sufficiently small $\Delta t > 0$ we can find a noise $v(\tau)$, $t \leqslant \tau \leqslant t + \Delta t$, such that by virtue of (1.5) $(x + \Delta x, q, t + \Delta t) \in D_k$ and $S_k^+(x, q, t) \leqslant S_k^+(x + \Delta x, q, t + \Delta t)$ for the corresponding increment Δx of the phase vector. In other words, if the controlling side in set D_k is slow in feeding in the impulse, then a noise exists which, in general, worsens the position from the point of view of minimizing the functional. By maximizing the right-hand side of the inequality presented with respect to v and letting $\Delta t \to 0$, we arrive at the inequality $P(S_k) = P(S_k^+) \geqslant 0$. Let us sketch the proof of the second inequality in (2.5). We assume the contrary: a point $N = (x, q, t) > \Gamma_k$ possessing property (2.6) and its ε-neighborhood N^ε with some $\varepsilon > 0$ exist for which the strict inequality $P(S_k^+) > 0$ holds when $(x, q, t) \in N^\varepsilon \cap G_k$. On the characteristics of the above-mentioned auxiliary optimal control problem we consider a point N_1 sufficiently close to N (Fig.1). Having assumed the contrary, we can find a noise value on the motion's segment from N_1 to Γ_k such that for the new point N_2 of going onto the boundary we obtain $S_k^+(N) \leqslant S_k^+(N_1) < S_k^+(N_2)$ in contradiction with the fact that from point N_1 the worst noise leads to the value $S_k^+(N)$ for functional J.

We note that if the quantity $P(S_k^+)$ is continuous in a neighborhood of Γ_k, then condition (2.5) yields the equality

$$P(S_k^+) = 0 \tag{2.7}$$

for the points $(x, q, t) \in \Gamma_k$, which for a known function S_k^+ can be treated as an implicit specification of surface Γ_k. Let us describe the algorithm for constructing the optimal functions S_k and u_k and the boundaries Γ_k with the aid of conditions (2.3)–(2.5) and (2.7).

Stage 1. The boundary Γ_k, the domain D_k, and the functions u_k and $S_k = S_k^+$ in this domain are known for some k. The boundary-value problem (2.3) is solved: $P(S_k) = 0$, $S_k = S_k^+$ when $(x, q, t) \in \Gamma_k$. The function $S_k(x, q, t)$ becomes known in the whole domain $D_k \cup G_k$ being examined. The values of the worst noise, furnishing the maximum with respect to v in operator P, are found when solving the boundary-value problem.

Stage 2. From the known function S_k, in accord with (2.1), the function S_{k+1}^+ is constructed for all (x, q, t), i.e., in the domain $G_{k+1} \cup D_{k+1}$. By the same token the functions S_{k+1} and u_{k+1} have been found in the as yet unknown domain D_{k+1} (see (2.4)). The quantity u_{k+1} supplies the minimum in (2.1).

Stage 3. Using the function S_{k+1}^+ constructed the boundary Γ_{k+1} is found by relations (2.5) and (2.7) as the set of points (x, q, t) on which the quantity $P(S_{k+1})$ changes sign, or vanishes. After this there is a return to Stage 1 with a number of impulses equal to $k + 1$.

The algorithm's work begins with Stage 1 for $k = 0$ with the $D_0 = \Gamma_0$ and $S_0^+ = F(x)$ specified in (2.3).

We remark on the application of relations (2.5) and (2.7). The finding of boundary Γ_1 qualitatively differs from the case of Γ_k, $k > 1$, because the minimum with respect to u in (2.1), for S_k^+, $k > 1$, in a neighborhood of points $(x, q, t) \in \Gamma_k$, is reached at an interior point u^*, $R(u^*) < q$. This is explained by the fact a nonzero resource must be left for the succeeding correction impulses. Therefore, when $k > 1$ we can expect the smoothness of function S_k^+ in a neighborhood of Γ_k and apply equality (2.7) for finding the boundary. It can be shown that under certain assumptions on functions f, F, R the surface Γ_1 lies on the boundary of the set of points (x, q, t) for which the minimum in (2.1) is reached on u^* such that $R(u^*) < q$. Thus, the nature of the minimum in (2.1) is different on the different sides of boundary Γ_1, which can be a source of nonsmoothness of function S_1^+ on Γ_1.

3. Optimal correction of a two-parameter system.

We consider the problem of designing the optimal correction for a system (1.1)—(1.4) of form

$$x^\cdot = (T-t)^\alpha u + v,\quad 0 \leqslant t \leqslant T,\quad |v| \leqslant 1,\quad u(t) = \sum_{k=1}^{N} u_k \delta(t-t_k),\quad \sum_{k=1}^{N} |u_k|^\sigma \leqslant Q,\quad J = |x(T)| \tag{3.1}$$

where x, u, v are vectors of like dimension, $|x|$ is the length of vector x, α, σ are positive scalar parameters. Here the function R from (1.3) has the form $R(u) = |u|^\sigma$. We note that by examining the equation of motion $x^\cdot = \varphi(T-t)u + \psi(T-t)v$ we can be led to form (3.1) if the scalar functions φ and ψ are related by $\varphi(h(\tau)) = a\tau^\alpha$ for some a and α, where $h(\tau)$ is the function inverse to $\rho(\xi)$ such that $d\rho / d\xi = \psi(\xi)$, $\rho(0) = 0$. The correction problem for the dynamic system $x^{\cdot\cdot} = u + v$ with functional $|x(T)|$ also can be reduced to form (3.1) with $\alpha = 1/2$.

Because system (3.1) is isotropic the Bellman function (1.7) for it will depend on three scalar arguments: $|x|$, q, $\tau = T - t$ and on an index k; the dimensions of the vectors make no difference to the analysis. Let us assume for simplicity that x, u, v are scalars. It can be shown that the function $S_k(|x|, q, \tau)$ for system (3.1) does not decrease with growth of $|x|$. Consequently, the minimum over u in (2.1) is reached when $u = -px / |x|$, $p = |u|$, and the minimization can be carried out with respect to parameter p. The transition from argument x to $|x|$ and from control u to p leads to the additional constraint $|x| - \tau^\alpha p \geqslant 0$. A consequence of the monotonicity of S_k with respect to $|x|$ is also the equality $|\partial S_k / \partial x| = \partial S_k / \partial |x|$. With due regard to these remarks we establish that the fundamental relations in (2.1)—(2.5), which are used to construct the optimal design, have the form

$$S_k^+(|x|, q, \tau) = \min_p S_{k-1}(|x| - \tau^\alpha p, q - p^\sigma, \tau),\quad S_k^+ = S_k,\quad p^\sigma \leqslant q,\quad |x| - \tau^\alpha p \geqslant 0 \tag{3.2}$$
$$P(S_k) = -\frac{\partial S_k}{\partial \tau} + \frac{\partial S_k}{\partial |x|} = 0,\quad P(S_k^+) \leqslant 0,\quad P(S_k^+) \geqslant 0$$

for system (3.1) and are invariant realtive to the one-parameter (with parameter C) group of transformations

$$S_k = CS_k',\quad S_k^+ = CS_k^{+\prime},\quad |x| = C|x'|,\quad \tau = C\tau',\quad q = C^{\sigma(1-\alpha)}q' \tag{3.3}$$

Therefore, we can seek the functions S_k and S_k^+, for example, in the form

$$S_k(|x|, q, \tau) = |x|\varphi_k(\xi, \eta),\quad S_k^+(|x|, q, \tau) = |x|\varphi_k^+(\xi, \eta),\quad \xi = \tau / |x|,\quad \eta = q|x|^{\sigma(\alpha-1)} \tag{3.4}$$

The boundaries Γ_k are curves in the plane of the self-similar variables ξ, η; these curves are obtained with the aid of operator P transformed to the new variables. Thus. the desired functions φ_k and φ_k^+ depend not on three but on two scalar variables. In domains G_k (in the variables ξ, η) we can obtain, by using (2.3), (3.2) and (3.4), the following equation for function φ_k:

$$P(\varphi_k) = -(1+\xi)\frac{\partial \varphi_k}{\partial \xi} + \sigma(\alpha - 1)\eta \frac{\partial \varphi_k}{\partial \eta} + \varphi_k = 0 \tag{3.5}$$

The general solution of this inhomogeneous linear equation /12/ has the form

$$\varphi_k(\xi, \eta) = (1+\xi)F_k(\eta(1+\xi)^{\sigma(\alpha-1)}),\quad k = 0, 1, \ldots, N \tag{3.6}$$

where F_k are functions of one variable.

Let us note a number of properties of the desired functions $F_k(\lambda)$. With a zero resource, i.e., with $q = 0$ $(\eta = 0)$, the Bellman function obviously is independent of index k. We have $\varphi_k(\xi, 0) = \varphi_0(\xi)$, whence, with due regard to the form of φ_0 obtained below we get $F_k(0) = 1$, $k = 1, \ldots, N$. In addition, $F_k(\lambda)$ are nonincreasing functions of argument λ, which follows from the corresponding property, noted in Sect.2, of the functions $S_k(x, q, t)$ with respect to argument q. The boundary Γ_0 is specified by the equality $\tau = T - t = 0$ or $\xi = 0$. Since for system (3.1) we have $J = F(x) = |x|$, from (1.8), (3.4) and (3.6) we obtain $\varphi_0(0, \eta) \equiv 1$ or $F_0 \equiv 1$. Thus, $\varphi_0 = \varphi_0(\xi) = 1 + \xi$ and Stage 1 of the algorithm in Sect.2 has been implemented. To implement the algorithm's second stage we write relation (2.1), with the aid of (3.2) and (3.4), in the form

$$\varphi_k^+(\xi, \eta) = \min_r h\varphi_{k-1}(\xi / h, \eta(1 - r^\sigma)h^{\sigma(\sigma-1)}),\quad 0 \leqslant r \leqslant 1,\quad h \geqslant 0,\quad r = p / q,\quad h = 1 - r\xi^\alpha\eta^{1/\sigma} \tag{3.7}$$

Using (3.7) for $k = 1$ and $\varphi_0 = 1 + \xi$, we obtain

$$\varphi_1^+(\xi, \eta) = \max[\xi,\ 1 + \xi - \xi^\alpha\eta^{1/\sigma}] \tag{3.8}$$

Two expressions for function φ_1^+ correspond to the interior and the boundary minimum over r in (3.7). By the remark made to Stage 3 in Sect.2 the boundary Γ_1 is a curve on which there expressions are equal. The same inference can be obtained by using (2.5), (3.5) and (3.8) since

$$P(\xi) = -1 < 0,\quad P(1 + \xi - \xi^\alpha\eta^{1/\sigma}) = \alpha\xi^{\alpha-1}\eta^{1/\sigma} \geqslant 0 \tag{3.9}$$

Equating the expressions within the brackets in (3.8) and using (2.5) and (3.9), we obtain

$$\Gamma_1 = \{(\xi,\eta):\ \xi^{\alpha\sigma}\eta = 1\},\quad D_1 = \{(\xi,\eta):\xi^{\alpha\sigma}\eta \leqslant 1\},\quad G_1 = \{(\xi,\eta):\ \xi^{\alpha\sigma}\eta > 1\} \tag{3.10}$$

By the same token we have taken one (the first) stage with respect to index k in the algorithm of Sect.2. According to Stage 1 of the next stage on boundary Γ_1 we have $\varphi_1 = \varphi_1^+$, whence with the aid of (3.6) and (3.8) we obtain an equality determining function F_1

$$F_1(\eta(1+\xi)^{\sigma(\alpha-1)}) = \xi/(1+\xi),\quad \xi^{\alpha\sigma}\eta = 1 \tag{3.11}$$

The effective implementation of Stage 2, i.e., the computation of the minimum in (3.7), depends essentially on the values of α and σ. From the results obtained for a programmed optimal correction problem /4,5/ it follows that in system (3.1) with $\sigma = 1$ each optimal correction impulse is directed opposite to vector x and either is a compensating impulse (making the current value of $|x|$ vanish) or uses up the whole correction resource. The positional form of such a correction law is

$$u_k = -q^{1/\sigma}x/|x|,\quad (x,q,\tau)\in D_1,\quad u_k = -x/\tau^{\alpha},\quad (x,q,\tau)\in D_k\setminus D_1,\quad k = 1,\ldots,N \tag{3.12}$$

in the initial variables and is

$$r_k = 1,\quad (\xi,\eta)\in D_1,\quad r_k = 1/(\xi^{\alpha}\eta^{1/\sigma}),\quad (\xi,\eta)\in D_k\setminus D_1,\quad k = 1,\ldots,N \tag{3.13}$$

in the selfsimilar variables (3.4) and (3.7). Thus, the values of (3.12) (of (3.13)) with $\sigma = 1$ furnish the minimum in (3.2) (in (3.7)).

Henceforth we restrict consideration to only those values of α and σ for which the design of the optimal correction in system (3.1) has the form (3.12), (3.13). Under certain values of parameters α and σ the optimal design, in general, can have another form. Under the assumption made an iterative application of the algorithm in Sect.2 leads to the following expressions for functions φ_k^+ and φ_k, $k = 1,\ldots,N$:

$$\varphi_k^+(\xi,\eta) = \begin{cases} 1+\xi-\xi^{\alpha}\eta^{1/\sigma}, & (\xi,\eta)\in D_1 \\ \xi F_{k-1}(\eta\xi^{\sigma(\alpha-1)}-\xi^{-\sigma}), & (\xi,\eta)\in G_1 \end{cases},\quad \varphi_k(\xi,\eta) = \begin{cases} \varphi_k^+(\xi,\eta), & (\xi,\eta)\in D_k \\ (1+\xi)F_k(\eta(1+\xi)^{\sigma(\alpha-1)}), & (\xi,\eta)\in G_k \end{cases} \tag{3.14}$$

and to a recurrent system of equalities for the construction of functions F_k and boundaries Γ_k

$$F_k(\eta(1+\xi)^{\sigma(\alpha-1)}) = \xi(1+\xi)^{-1}F_{k-1}((\rho-1)/\xi^{\sigma}),\quad (\xi,\eta)\in\Gamma_k,\quad k = 1,\ldots,N \tag{3.15}$$

$$F_{k-1}((\rho-1)/\xi^{\sigma}) + \sigma\xi^{-\sigma}[1+\xi+(\alpha-1)\rho]F_{k-1}'((\rho-1)/\xi^{\sigma}) = 0,\quad k = 2,\ldots,N,\quad \rho = \xi^{\alpha\sigma}\eta \tag{3.16}$$

Equality (3.15) is a consequence of boundary condition (2.3), obtained by use of (3.4) and (3.6); for known Γ_k and F_{k-1} it determines the function F_k. Differential equality (3.16) is condition (2.7) of the explicit specification of boundaries Γ_k, transformed by (3.5) and (3.6); for a specified F_{k-1} it serves to find the points of boundary Γ_k. If we have $F_0 \equiv 1$ and Γ_1 in (3.10), we can use (3.15) and (3.16) to determine in succession $F_1, \Gamma_2, F_2, \Gamma_3, \ldots, F_k, \Gamma_{k+1}, \ldots$. After this the domains D_k and G_k become known, and the required quantities S_k and u_k are determined by equalities (3.4), (3.14), (3.12).

Thus, in the case being examined the solution of the design problem for the optimal correction of system (3.1) reduces to the construction of two families of curves by using relations (3.15) and (3.16): the boundaries Γ_k and the graphs of functions $F_k(\lambda)$. When $\alpha = 1$ relations (3.15) and (3.16) simplify somewhat and admit of the following analytic solution:

$$F_k(\eta) = [1+(\eta/k)^{1/\sigma}]^{-k},\quad \Gamma_k = \{(\xi,\eta):\ \xi^{\sigma}\eta = k\},\quad k = 1,\ldots,N \tag{3.17}$$

When $\sigma = 1$ formulas (3.17) can be obtained as well from the solution of the programmed problem considered in /3,5/.

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